Linear Algebra Date : April 17, 2020
Time : 13.45 – 15.45 hrs, upload before 16:00 hrs.
Students with extra time must upload before 16:30 hrs.

First read these instructions carefully:

You make the exam on paper at home. At the end of the exam you take pictures of your exam and create a single PDF file containing your exam and you upload the exam onto Canvas. Obviously you can also write on an iPad or type it in LaTeX or Word (the latter is likely to be too time-consuming but allowed). Include your name on the first page and in the filename of your document.
The results of this exam are preliminary and will only become permanent when we are convinced the big majority of you deserve to obtain this grade. We will perform some short oral exams on a random selection of students to check whether your understanding of the material is in line with the work that was handed in.
You have to make this exam by yourself without the help of a calculator or computer software. However, you can use the lecture notes and your own notes while making the exam. In all questions include clear arguments and your calculations.

1. Please read the following paragraph carefully, and copy the text below it verbatim to your answer sheet. By testing you remotely in this fashion, we express our trust that you will adhere to the ethical standard of behaviour expected of you. This means that we trust you to answer the questions and perform the assignments in this test to the best of your own ability, without seeking or accepting the help of any source that is not explicitly allowed by the conditions of this test.

   Text to be copied: *I will make this test to the best of my own ability, without seeking or accepting the help of any source not explicitly allowed by the conditions of the test.*

2. Given are two matrices:

$$A = \begin{pmatrix} 1 & 1 & \alpha \\ 1 & \beta & 2 \\ 1 & 0 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

   It is known that B is an echelon form of the matrix A. Determine α and β.

3. We know

$$\begin{pmatrix} 1 \\ 3 \\ \alpha \end{pmatrix} \in \text{Span}\left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix} \right\}$$

   Determine α.

4. Given is that $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is a basis of the subspace $\mathcal{V}$. Show that $\{\mathbf{x}_1, \mathbf{x}_1 + \mathbf{x}_2, \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3\}$ is also a basis of the subspace $\mathcal{V}$.

5. Given are matrices A and B:

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 1 \\ 2 & -1 \end{pmatrix}$$

   Find all matrices X such that $AX = B$.

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6. Given is the matrix

$$A = \begin{pmatrix} 1 & 2 & \alpha \\ 1 & \alpha & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

with $\alpha \in \mathbb{R}$.

a) Find all $\alpha \in \mathbb{R}$ for which

$$\text{Null}\, A = \text{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$

b) Find all $\alpha \in \mathbb{R}$ for which

$$\text{Col}\, A = \text{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

7. Given is that for the matrix

$$A = \begin{pmatrix} 3 & -2 & -2 \\ -1 & 1 & 2 \\ -4 & 3 & \alpha \end{pmatrix}$$

the inverse is given by:

$$A^{-1} = \tfrac{1}{3} \begin{pmatrix} 1 & 8 & -2 \\ -1 & 13 & -4 \\ 1 & -1 & 1 \end{pmatrix}$$

Determine α.

8. Given is the matrix

$$A = \begin{pmatrix} 5 & -1 & 3 & -1 \\ 3 & 1 & 3 & -1 \\ 2 & 2 & 3 & -1 \\ -3 & -4 & 1 & 8 \end{pmatrix}$$

Find all α for which

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ \alpha \end{pmatrix}$$

is an eigenvector of the matrix A.

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9. Given is a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that:

$$T\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}, \qquad T^2\begin{pmatrix}1\\2\end{pmatrix} = 2\begin{pmatrix}2\\1\end{pmatrix}.$$

a) Determine the representation matrix of T.

b) Determine whether the transformation T is onto.

Problem:	2	3	4	5	6	7	8	9
points:	5	5	6	5	4+4	5	5	3+3

Maximal number of points p is 45. The preliminary exam grade is $(p + 5)/5$ if question 1 is answered appropriately. Otherwise the grade is 1.