Mathematics C1 (Cayley)

Date : February 20, 2015
Time : 13.45 – 14.45 hrs

The solutions to the exercises need to be wellstructured and clearly formulated.
Moreover, you need to motivate your answer in all cases!
The use of electronic devices is not allowed.

1. The matrix A and the vectors **b** and **p** are given by:

$$A = \begin{bmatrix} 2 & 3 & -2 & -4 \\ -1 & 2 & 8 & -5 \\ 1 & 1 & -2 & -1 \end{bmatrix}; \quad \mathbf{b} = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}, \quad \mathbf{p} = \begin{bmatrix} 2 \\ 9 \\ 2 \\ 5 \end{bmatrix}$$

(a) [1 pt] Show that **p** is a solution of the system $A\mathbf{x} = \mathbf{b}$.

(b) [2 pt] Determine the solution set of the system $A\mathbf{x} = \mathbf{0}$ and write it in parametric vector form.

(c) [2 pt] Determine the solution set of the system $A\mathbf{x} = \mathbf{b}$ and write it in parametric vector form.

(d) [2 pt] Determine all solutions of $A\mathbf{x} = \mathbf{b}$ for which $x_2 = 0$.

(e) [1 pt] Express, if possible, **b** as a linear combination of the columns of A.

2. (a) [2 pt] Let $A = \begin{bmatrix} -2 & \beta \\ \alpha & 2 \end{bmatrix}$. Determine all integers $\alpha, \beta \in \mathbb{Z}$ for which $A^2 = I_2$.

(b) [2 pt] Determine the inverse of the matrix $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 5 & 3 \end{bmatrix}$, if it exists.

(c) [2 pt] Let $C \in \mathbb{R}^{n\times n}$ be a matrix and let $\mathbf{b} \in \mathbb{R}^n$ such that the system $C\mathbf{x} = \mathbf{b}$ is inconsistent. Is it possible that there exists $\mathbf{c} \in \mathbb{R}^n$ such that the system $C\mathbf{x} = \mathbf{c}$ has exactly one solution? Explain your answer by using the echelon form of C.

3. Let $A \in \mathbb{R}^{n\times n}$ be an invertible matrix.

(a) [2 pt] Show that $\mathbf{y} = A^{-1}\mathbf{b}$ is a solution of the system $A\mathbf{x} = \mathbf{b}$.

(b) [2 pt] Show that if **y** is a solution of the system $A\mathbf{x} = \mathbf{b}$ then necessarily $\mathbf{y} = A^{-1}\mathbf{b}$.

Total: 18 points