Test Probability Theory, 1 July 2019, 13:45-15:45h
Module Data & Information (201300180)

This test has 6 exercises, a formula sheet and tables of the binomial and normal distribution.
You should motivate all your answers.
A simple scientific calculator is allowed, a graphical/programmable calculator is not allowed.

1. For each of the following statements, mention if it is true or not (and motivate why).

   (a) If two random variables X and Y are uncorrelated, then they are independent.

   (b) If X is the number of colour-blind people in a random sample of 250 Dutchmen and 2% of the Dutch are colour-blind, then X is approximately Poisson distributed.

2. A (female) football player is just 2 goals away from becoming her nation's all-time leading markswoman with 60 goals for the national team. She hopes to achieve this milestone in the upcoming match.

   Let N be her number of attempts on goal in a match. Assume N is Poisson distributed with parameter 6. Each of her attempts on goal has probability 0.3 of becoming a goal, and these attempts are independent. Let X be her number of goals in the match.

   (a) If there are 7 attempts on goal, what is the probability distribution of X and what is its expectation?

   (b) Determine $P(X = 2 \text{ and } N = 7)$.

   (c) How should we calculate $P(X = 2)$? Your answer may be an arithmetic expression, showing how to calculate this. (You need not calculate this probability.)

3. High school students who need to pass a test can either choose to study hard for the test or not. Suppose that 80% of the students who study hard pass the test, while only 10% of the remaining students pass the test. Given that 60% of all students chooses to study hard (and 40% not), determine the probability that a student who passed the test has studied hard. First, define appropriate events and use these to answer the question.

4. An activity is organized for members of Inter-Actief. One estimates from past experience that each member will participate with probability 0.8 (independent of one another).

   (a) What is the probability distribution of the number of participating members (name and parameters)?

   (b) There are 6 board members. What is the probability that 5 or 6 of them participate?

   (c) Inter-Actief has (approximately) 1000 members. Calculate or approximate the probability that 780 or more Inter-Actief members participate in the activity.

5. The time T (in seconds) for the completion of two sequential events is a continuous random variable with probability density function

$$f_T(t) = \begin{cases} \frac{c}{t^3}, & 1 \leq t \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Show that $c = \frac{32}{15}$.

   (b) Calculate $E(T)$, $E(T^2)$ and $var(T)$.

   (c) What is the probability that the events are completed in no more than 3 seconds?

   (d) What is the density function of $Y = \sqrt{T}$?

6. A cereal manufacturer has a machine that fills boxes. Boxes are labeled "450 grams". Since no packaging process is perfect, there will be minor variations. Based on the experience with the packaging machine, the company believes the amount of cereal in a box fits a normal model with a standard deviation of 6 gram (g). The manufacturer decides to set the machine to put an average of 460 g in each box.

   (a) What is the probability that a box will be underweight (weighs less than 450 g)?

   (b) The company's lawyers insist that no more than 3% of the boxes can be underweight. What mean setting (instead of 460 g) should the company use to achieve this?

   (c) Consider a random sample of 10 cereal boxes (when the machine puts an average of 460 g in each box). What is the probability that their total weight is between 4590 and 4610 g? Which assumptions do you need to make?

Points:

1		2			3	4			5				6			total
a	b	a	b	c		a	b	c	a	b	c	d	a	b	c	
1	1	2	2	3	3	1	2	3	1	3	2	2	2	2	3	33

Grade: $\dfrac{\text{number of points}}{33} \times 9 + 1$

Formula sheet Probability Theory for BIT and TCS in module 4

Distribution	$\boldsymbol{E(X)}$	$\boldsymbol{var(X)}$
Geometric	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Hypergeometric	$n \cdot \frac{R}{N}$	$n \cdot \frac{R}{N} \cdot \frac{N-R}{N} \cdot \frac{N-n}{N-1}$
Poisson $P(X = x) = \frac{e^{-\mu}\mu^x}{x!},\ x = 0, 1, 2, \ldots$	μ	μ
Uniform on $[a, b]$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Exponential	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Erlang $f_X(x) = \frac{\lambda(\lambda x)^{n-1}e^{-\lambda x}}{(n-1)!},\ x \geq 0$	$\frac{n}{\lambda}$	$\frac{n}{\lambda^2}$

$$var\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} var(X_i) + \sum\sum_{i \neq j} cov(X_i, X_j)$$